Parallel computing in terms of Gaisi Takeuti's quantum set theory[1]

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Abstract: The foundations and results of parallel computing in terms of Gaisi Takeuris's quantum set theory [1] are presented in this review paper. This approach is based on 1) the extension of the von Neumann's uniqueness theorem for the solutions of the canonical commutations relations in quntum mechanics of finitely many degrees of freedom to quantum local field theories of infinitely many degrees of freedom [2]; 2) the results of the mathematician Gaisi Takeuti about quantum set theory in extension theory; and the results of the physicist Lev Davidovics Landau about the behavior of the quantum liquids He^3 and He^4 resulting in a more general approach than the one base on the notion of the quantum bit [3, 4, 5].

Keywords: quantum logic, ZFC axioms, set theory, quantum bit, computing, Haag-theorem, quantum computer, local Hamiltonian, Hilbert bundle, binary numbers, quantum liquids

1. Introduction

An alternative quantization method of local field theory (LFT) was proposed four decades ago in the papers [2, 6]. This approach is a direct extension of the canonical quantization method of Quantum Mechanics (QM) of finitely many degrees of freedom to Quantum Field Theory (QFT) of infinitely many degrees of freedom. It preserves the structure of LFT, namely the global observables of the system are generated by local ones integrated them over the underlying space-time structure. Since a classical local field theory is built up from an infinite collection of identical classical mechanical systems of finitely many degrees of freedom connected in space, we expect that the quantization preserves this structure. In fact, generally one can say that this quantization method substitutes the individual members of the infinite collection by their quantum mechanical refinements connected in space.
In this way an alternative approach of quantized fields was constructed which applies a new (quantum) conception of space-time, in accordance with Schwinger's observation ([7], see below point B)) and it lacks the main difficulties of the conventional theory. These difficulties are as follow:

1A) If the basic hypotheses of the conventional theory hold true then the interaction picture is not applicable to describe nontrivial interactions. This is stemmed by two difficulties: a) in the framework of the conventional theory, by means of the interaction picture, one can derive only the trivial S matrix (Haag´s theorem), and b) the interaction Hamiltonian consisting of higher power than quadratic of the field does not possess a definite mathematical meaning in the Fock space of the free field (ultraviolet catastrophe).
B) The quantum fields defined at the points of the Minkowski space do not exist as operators in a Hilbert space.

In the alternative approach the local bounded observables of the system are represented by bounded self-adjoint operators (A-module homomorphism) in the local state space H_A where H_A is an A-valued Hilbert space (Hilbert A-module) and A is the algebra of bounded operators in the Hilbert-

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space $L^2(\boldsymbol{R}^3)$ (the Hilbert-space of the square integrable functions over the space $\boldsymbol{R}^3$). The local states are represented by the rays of norm *1* (the unity operator of A) in H_A. The expectation value of a local bounded observable F in the local state Ψ in H_A is given by the formula

$$Exp\ F = <\Psi|F|\Psi>_A \quad \in \quad A \qquad (1.1)$$

useing the A-valued Hermitian inner product of H_A.

The global description of quantum local field theory (QLFT) of local state space H_A corresponds to the rules:

a1) The global state space of QLFT of local state space H_A in the measuring procedures characterized by the statistical operator $\rho \in A$ is the complex separable Hilbert space $H^\rho = \mathrm{Tr}\ \rho H_A$; its global states are described by the rays of H^ρ.

a2) The global observables are represented by self-adjoint operators in H^ρ.

a3) The expectation value of the global observable f generated by the local one, F, in the global state ϕ generated by the local one, Φ, is

$$f_\rho = <\phi|f|\phi>_\rho = \mathrm{Tr}\ \rho<\Phi|F|\Phi>_A , \qquad \Phi \in \boldsymbol{\Phi} \qquad (1.2)$$

H^ρ carries all the information on the infinite collection of connected quantum mechanical systems constituting QLFT, which are obtained in the common quantum statistical state ρ of the local measuring apparatuses in (quantum) space-time.
We note that it was demonstrated in the paper [8] that this approach of QLFT does contain the physical implications of the conventional theory legitimating in this way this alternative quantization method of local fields.

1B) Let L denotes the lattice of all closed linear subspaces of a Hilbert space H. By John von Neumann [9] it is called quantum logic, the intrinsic logic of the quantum world. Gaisi Takeuti [1] showed that set theory based on von Neumann´s quantum logic (named quantum set theory) satisfies the generalization of the ZFC axioms (Zermelo, Frenkel plus Axiom of Choice) of set theory. Therefore a reasonable mathematics can be derived from quantum set theory but a much richer mathematics, a „gigantic" mathematics by the words of Takeuti.

In the paper [2] the basic mathematical problems of QFT quoted above were collected and a way out of these difficulties, based on by repeating von Neumann´s line of thoughts for finitely many degrees of freedom, i. e. for QM, was outlined.
The two main points of von Neumann are here:

a) He observed [10] that both of Schrödinger´s wave mechanics and Heisenberg´s matrix mechanics can be built up from so called elementary observables ("propositions") taking the values 0 and 1, since both the Schrödinger´s differential operators and the Heisenberg´s matrices are operators acting on Hilbert spaces and one can spectrally build up them from orthogonal projectors representing these elementary observables in the corresponding Hilbert space.

b) The canonical commutation relations have unique solutions up to unitarily equivalence, i. e. if one chooses either Schrödinger´s wave mechanics or Heisenberg´s matrix mechanics one gets the same results. The two representations are unitarily equivalent (**von Neumann´s theorem**). (This

theorem guarantees, roughly speaking, the stability of quantum theory for finitely many degrees of freedom.)

For infinitely many degrees of freedom an alternative solution was proposed four decades ago.

1BA) The elementary propositional systems of local field theories were investigated and found that these propositions can not only take the values 0 and 1 but they have (infinitely many) third values, too, the so called **true-false** values. Thus, in the case of systems with infinitely many degrees of freedom, von Neumann´s line of thoughts steps beyond the mathematics based on the two valued logic [2, 11].

1BB) The representations of the elementary propositional systems were looked for and the solutions of the commutation relations were studied on these representations. For this reason one had to turn to the extension of the basic tools of the theory of Hilbert spaces. Then it was found that the extended form of the von Neumann´s theorem holds true on these representations [2)]. (So in this way the quantization of local fields could lead, roughly speaking, to a stable quantum field theory [2].)

This alternative solution of quantized fields with infinitely many degrees of freedom uses (based on) the „gigantic" mathematics derivable from the quantum set theory of Takeuti [1, 2].

1C) It is a natural question that what is about computing in this framework? We discussed this question in the papers [3, 4, 5]. We concluded that a) computing based on quantum set theory offers a more general framework than the one based on the notion of the quantum bit, and as a corollary b) it could and should offer a computing machinery exceeding the capacity of the computers we are using in these decades. The main points in these studies are as follow:

1CA) As we mentioned we investigated the elementary propositional systems of local field theories in the paper [11] and found that these propositions can not only take the values 0 and 1 but they have (infinitely many) third values, too, the so called **true-false** values. Thus again, in the case of systems with infinitely many degrees of freedom, von Neumann´s line of thoughts steps beyond the mathematics based on the two valued logic.

1CB) In accordance with the point 1BB), the representations of the elementary propositional systems were looked for and the solutions of the commutation relations were studied on these representations. For this reason we had to turn to the ***extension*** of the basic tools of the theory of Hilbert spaces, to the theory of Hilbert A-modules H_A, where A is the C*-algebra of operators in the Hilbert-space $L^2(\mathbf{R}^3)$. Then it was found that the extended form of the von Neumann´s theorem holds true on these representations [2]. We call the representation space H_A as *the local state space of the quantized system.*

1CC) We showed in [8] that 1) this alternative solution of quantized fields with infinitely many degrees of freedom reproduces the physical implications of the conventional theory, legitimating in this way the alternative approach (cf. [12]), and in [2, 13] that 2) it uses (based on) the „gigantic" mathematics derivable from the quantum set theory of Takeuti [1].

1CD) One can find the illustration of the geometrical structure of the system´s local state space both in references [13, p. 1059] and [2, p. 198]. It shows that one may think of this structure as a „non commutative" Hilbert bundle. Therefore the conclusion is that the local states of the system [*consisting of an infinite collection of (identical) quantum systems of finitely many degrees of*

freedom connected in space] are sections of the bundle. The time evolution of these local states is governed, instead of the global/total Hamiltonian, by the **local Hamiltonian** of the system according to the eq. (30) in ref. [13] or to the eq.s (5.8a) and (5.8b) in ref. [2]. This geometrical structure and time evaluation equations implies that the different alternatives [*for the individual members of the infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space*] given by an initial value of the evolution equation described by a section of the „non commutative" Hilbert bundle can be computed in *parallel* [3].

1CE) In the paper [14] we discussed further on this theoretical possibility by using an explicit example of a rigid body of cuboid form. The universe $V^{(L)}$ of Takeuti was determined. A set of real numbers in this universe was explicitly described including a set of binary numbers. Thus we arrived at the foundations of von Neumann's theory of computing in terms of ordinary binary numbers. Then we concluded that this ***extension*** of computing to the universe $V^{(L)}$ provides a sound, mathematically well-defined theory of quantum computing.
In the paper [4] we discuss this theoretical possibility further on by studying its mathematical apparatus for the illustrative case of N real classically relativistic scalar fields of Bose-Einstein statistics together with the solution of the eigenvalue problem for the free field approximation and the possible application of the results in computing.
Then in the paper [5] we discussed in this approach to parallel computing, in details, the specific case of the fermions of spin ½ by using the Dirac equation. In these two papers [4, 5] the quantum liquids He^3 of Fermi statistics and He^4 of Bose-Einstein statistics described by L. D. Landau at low temperature (below 3 K) using the quasiparticle approach were studied and discussed, in the framework of the alternative field quantization [2], how these physical materials may be candidates for building the processors of computers of type based on the quantum set theory of G. Takeuti [1].

In section 2 the extension of the von Neumann's theorem is presented. Then section 3 and section 4 are devoted to summarize the technical details, including the solution of the eigenvalue problem for the free field approximation, for the quantization of the local scalar fields of Bose-Einstein statistics and local spinor fields of Fermi statistics, respectively, and the possible applications of these results in parallel computing. Section 5 closes this paper with conclusions. For the sake of simplicity we use the natural units $c = h/2\pi = 1$ in this paper.

2. The extension of the von Neumann's theorem

Now we formulate the canonical quantization of classical LFT (CLFT) of Lagrangian density

$$L(t,\boldsymbol{x}) = [1/2\Sigma_{\alpha=1}{}^{N}(\partial_\mu\phi^\alpha\partial_\mu\phi^\alpha - m_\alpha{}^2\phi_\alpha{}^2) - V(\phi_1, \ldots, \phi_N)](t,\boldsymbol{x}),\ \ (t,\boldsymbol{x})\in\boldsymbol{M}^4 \qquad (2.1)$$

in terms of the local state space H_A and its unbounded extension $H_{\hat{A}}$, where $\hat{A}$ is the unbounded extension of the C*-algebra of the bounded operators in the Hilbert space $L^2(\boldsymbol{R}^3)$ of square integrable functions over $\boldsymbol{R}^3$. In general in what follow the unbounded extension of a mathematical object in a specific model of set theory is denoted by a hat.
One sees that this system of Lagrangian density (2.1) consists of an infinite collection of identical classical anharmonic oscillators of N degrees of freedom connected in space.
One postulates that the variables ϕ_α, π_α, $\partial\phi_\alpha$, $\partial\pi_\alpha$, $\alpha = 1, \ldots, N$, of CLFT are local observables and thus represented by self-adjoint operators in the unbounded extension of the algebra of bounded operators $B(H_A) = B(H)\otimes A$ in the local state space H_A [2]. Furthermore the local fields ϕ_α and momentums π_α satisfy the equal time CCR's

$$[\phi_\alpha, \phi_\beta] = [\pi_\alpha, \pi_\beta] = 0, [\pi_\alpha, \phi_\beta] = -i\delta_{\alpha\beta}\mathbf{1}, \quad \alpha, \beta = 1, \ldots, N \qquad (2.2)$$

where $\mathbf{1} = 1_H \otimes 1$, $1 \in A$, and equalities are of course understood on the common domain of the both sides. An example for operators satisfying (2.2) is the following. Let $\{\varphi_n\}^{\infty}_{n=1}$ be a basis in $L^2(\boldsymbol{R}^3)$, then

$$\phi_\alpha = \Sigma_n q_\alpha \otimes c_n^{\alpha} |\varphi_n\rangle\langle\varphi_n|, \quad \pi_\alpha = \Sigma_n p_\alpha \otimes (c_n^{\alpha})^{-1} |\varphi_n\rangle\langle\varphi_n|, \tag{2.3a}$$

where

$$[p_\alpha, p_\beta] = [q_\alpha, q_\beta] = 0, \quad [p_\alpha, q_\beta] = -i\delta_{\alpha\beta} 1_H \tag{2.3b}$$

and $c_n^{\alpha} \in \mathbf{R}$, while p_α, q_β are elements of the unbounded extension of $B(H)$, and $\alpha, \beta = 1, \ldots, N$. By using the Poisson brackets of the basic local variables ϕ *and* π and their gradients $\partial\phi$ and $\partial\pi$, furthermore from the irreducibility requirement for the system of basic variables (ϕ, π) at each point $\mathbf{x} \in \boldsymbol{R}^3$ and with the notation $\mathbf{p} = -i\partial_{\mathbf{x}}$ we derived the relations for the observables in the local state space in H_A that

$$\partial\phi = \mathbf{p}\phi = \phi\mathbf{p}, \qquad \partial\pi = \mathbf{p}\pi = \pi\mathbf{p} \tag{2.4}$$

Since

$$[p_1 \otimes a_1, p_2 \otimes a_2] = p_1 p_2 \otimes [a_1, a_2] + [p_1, p_2] \otimes a_2 a_1 = p_2 p_1 \otimes [a_1, a_2] + [p_1, p_2] \otimes a_1 a_2$$

where $a_1, a_2 \in \hat{A}$ and p_1, p_2 are elements of the unbounded extension of $B(H)$, with (2.4) we obtain the general form of the basic observables ϕ *and* π. They are

$$\phi_\alpha = \phi_\alpha(\mathbf{p}) = q_\alpha \otimes c_\alpha, \quad \pi_\alpha = \pi_\alpha(\mathbf{p}) = p_\alpha \otimes c_\alpha^{-1}, \quad c_\alpha = c_\alpha^*, \tag{2.5}$$

where c_α's are invertible elements of the unbounded extension of the Abelian von Neumann algebra B generated by a complete Boolean sublattice of the lattice of all orthogonal projectors in the Hilbert space $L^2(\boldsymbol{R}^3)$ (B is a subalgebra of A). Furthermore $(p, q) = (p_1, \ldots, p_N, q_1, \ldots, q_N)$ is an irreducible system of self-adjoint operators in H satisfying the CCR's (2.3b). A well-known solution for (p, q) is

$$H = L^2(\boldsymbol{R}^N), \quad q_\alpha = q_\alpha \cdot, \quad p_\alpha = -i(\partial/\partial q_\alpha), \quad \alpha = 1, \ldots, N \tag{2.6}$$

Let us cast the CCR's (2.2) into Weyl's form. We define the unitary operators

$$U(\alpha) = \exp\{i\Sigma_{n=1}^{N} \alpha_n \phi_n\}, \; V(\alpha) = \exp\{i\Sigma_{n=1}^{N} \alpha_n \pi_n\}$$

in H_A, where $\alpha = (\alpha_1, \ldots, \alpha_N) \in \boldsymbol{R}^N$. We assume that U and V are continuous with respect to the parameters α and satisfy the relations

$$U(\alpha)U(\beta) = U(\alpha + \beta), \; V(\alpha)V(\beta) = V(\alpha + \beta), \; U(\alpha)V(\beta) = \exp\{i\Sigma_{n=1}^{N} \alpha_n \beta_n\} V(\beta)U(\alpha). \tag{2.7}$$

Definition 2.1: A system (U, V) of bounded operators in the A-valued Hilbert space H_A is called β-irreducible system if the set of bounded operators in H_A commuting with all the members of the system (U, V) is equal to the Abelian von Neumann algebra B generated by the complete Boolean sublattice β from the lattice of the orthogonal projertors generating the C*-algebra A.

We note that this irreducibility notion is the natural extension of the irreducibility notion formulated in the β-valued universe $V^{(\beta)}$ to the L-valued universe $V^{(L)}$ where L is the lattice of the all orthogonal projectors of a Hilbert space H, i.e $V^{(L)}$ is the universe of G. Takeuti's quantum set theory [1]. For the extension of the irreducibility notion in the ordinary universe V to the irreducibiliy notion in $V^{(\beta)}$ is as follows: A system of bounded operators in the Hilbert space $H^{(\beta)}$ in $V^{(\beta)}$ is irreducible if its center is equal to the bounded part of the complex numbers $C^{(\beta)}$ in $V^{(\beta)}$, i.e. it is equal to B (cf. [15]).

Then we have the following extension of von Neumann's theorem.

Proposition 2.2: A β-irreducible set of unitary operators $U(\alpha)$ and $V(\alpha)$ in the A-valued Hilbert space H_A satisfying the CCR's (2.7) is uniquely determined up to A-unitary equivalence, i.e., if $(U'(\alpha), V'(\alpha))$ is another set of β-irreducible unitary operators in H'_A also satisfying (2.7), then an A-unitary operator $S: H'_A \rightarrow H_A$ exists such that $U(\alpha) = SU'(\alpha)S^{-1}$ and $V(\alpha) = SV'(\alpha)S^{-1}$.

Proof: We apply the methods of β- and L-valued models for the proof of the proposition. Here $H_A = H\otimes A$ is the bounded part of the Hilbert space $H^{(L)} = H\otimes \hat{A}$ in $V^{(L)}$ and $H'_A = H'\otimes A$ is the bounded part of the Hilbert $H'^{(L)} = H'\otimes \hat{A}$ in $V^{(L)}$. Furthermore, it follows from the β-irreducibility property of the two set $(U(\alpha), V(\alpha))$ and $(U'(\alpha), V'(\alpha))$ that $U(\alpha), V(\alpha) \in B(H_B) = B(H^{(\beta)})$ is in $B(H_A)$ and $U'(\alpha), V'(\alpha) \in B(H'_B) = B(H'^{(\beta)})$ is in $B(H'_A)$ [$U(\alpha)$ and $V(\alpha)$ map the subspace $H_B = H\otimes B$ of H_A onto itself and $U'(\alpha)$ and $V'(\alpha)$ map the subspace $H'_B = H'\otimes B$ of H'_A onto itself]. In this way the system $(U(\alpha), V(\alpha))$ is an irreducible systems of unitary operators in the Hilbert space $H^{(\beta)}$ in $V^{(\beta)}$ and the system $(U'(\alpha), V'(\alpha))$ is an irreducible systems of unitary operators in the Hilbert space $H'^{(\beta)}$ in $V^{(\beta)}$. Now, because $V^{(\beta)}$ satisfies the axioms ZFC [16] we can apply theorem 30 in Ref. [16] (pp. 55-57) to von Neumann's theorem in the ordinary universe V which yields that von Neumann's theorem also holds true in $V^{(\beta)}$ [i.e., $(U(\alpha), V(\alpha))$ in $H^{(\beta)}$ and $(U'(\alpha), V'(\alpha))$ in $H'^{(\beta)}$ are unitarily equivalent in $V^{(\beta)}$]. Then the canonical extension of the corresponding unitary operators in between $H^{(\beta)}$ and $H'^{(\beta)}$ provides the A-unitary operator S in between H_A and H'_A. Q.E.D.

This extension of von Neumann's theorem offers the possibility that we formulate QLFT in terms of the A-valued Hilbert spaces in the same unique way, up to A-unitary equivalence as QM is formulated in terms of complex Hilbert spaces up to unitary equivalence. As we pointed out in Ref. [13] one can consider the representation space H_A as the local state space of QLFT constructed over quantum space-time of event space $L^2(\boldsymbol{R}^3)$ ("local" here means the locality in quantum space-time [17, 6]. One can then straightforwardly adapt the "rules" of the Hilbert space formulation of QM to QLFT [2].

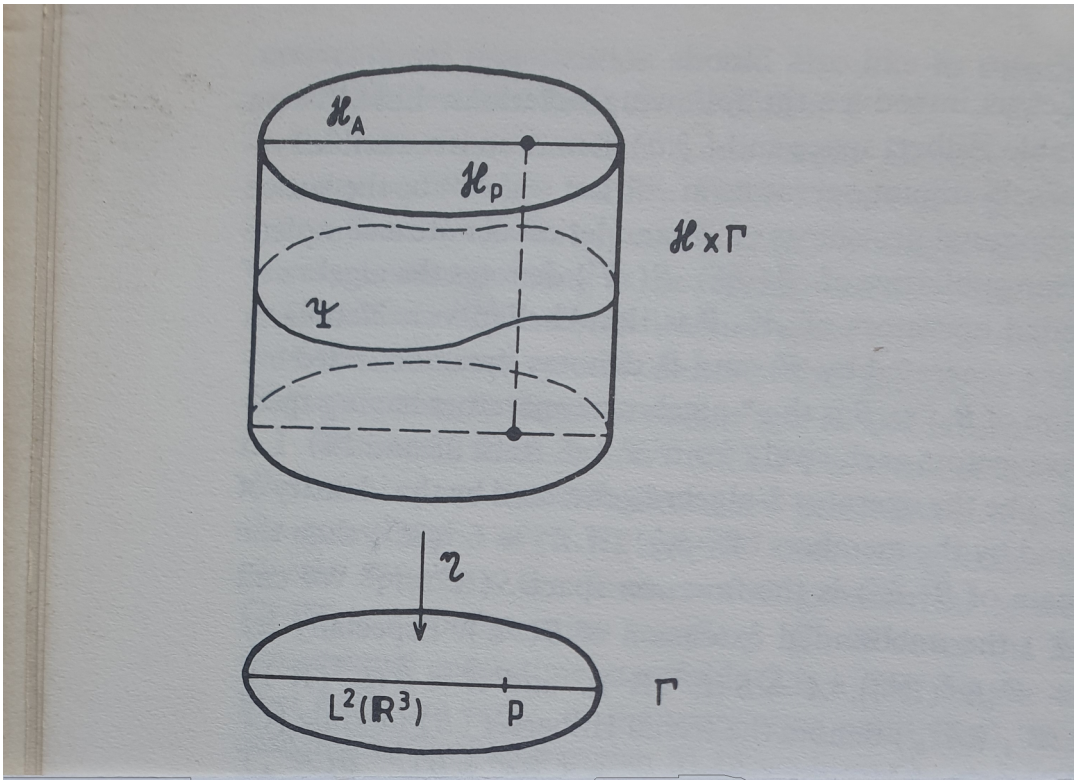
In the next step let we illustrate geometrically the local state space H_A.

We start with a remark. We have a problem to be solved. We have to study 100 diferent alternatives to obtain the solution. Our computers in these days are able to compute the 100 alternatives consecutively. The professionals expect from the „quantum computer" that it will be able to sudy the 100 alternatives in parallel, simultaneously. In this way it reduces the time of the solution radically.

How?

We can illustrate the geometrical picture of the caracteristic mathematical objects in the quantum set theory of Gaisi Takeuti by the next figure (Figure 1.). One can see in the figure that the elementary systems constituting the whole system are side by side in space. The collective states of the system are given by the sections of the compact cylinder which are evolving thogether (parallel) in time. The appropriate mathematical object, the local state space of the system describes the system **coherently (not in a loosely way)**. Γ represents the event space of the underlying spacetime, $H^x\Gamma$

over it is representing the local state space H_A of the system with infinitely many degrees of freedom. At the event p there is a state Hilbert space H_p describing the states of the elementary system of finitely many degrees of freedom at the event/point of the underlying space-time. Thus one can think of the local state space as a kind of a Hilbert bundle. The section Ψ represents a local state of the whole system.



Geometrical illustration of the local state space H_A.

Figure 1.

To close this section we discuss computing in term of quantum set theory [1].

2.1. *Remarks about quantum computing*:

2.1.1 The classical bit (0, 1) is an observable. One can derive all observables from them as elementary observables (considering quantum systems with finitely many degrees of freedom and arbitrary classical systems) [11]. However the quantum bit (0, superpositions, 1) is not an observable but a two dimensional state space, thus one could directly not derive all observables from them.

2.1.2 *Conjecture:* The quantum bits generate a state space with a Fock space structure, therefore, as we guess, this approach might knock against the Haag-theorem [2, 6, 8], i. e. it might not be able to describe, in a mathematically rigorous way, interacting fields only free fields.

2.1.3 Then one could conclude from points 2.1.1 and 2.1.2 that the concept of the quantum bit may be incomplete. While the classical bit satisfies the completeness criterion, i. e. all observables can be derived from them (in the cases of arbitrary classical systems and quantum systems with finitely many degrees of freedom), the quantum bit does not satisfy the completeness criterion: one could

not derive all observables from them in the cases of quantum systems with infinitely many degrees of freedom.

2.2. C*omputing based on quantum set theory*:

2.2.1 One can build this approach on the quantum logic of von Neumann [9] and on the quantum set theory of Takeuti [1] instead of the notion of the quantum bit. This approach generalizes the real numbers. It turns from mathematics based on the two valued logic to mathematics based on the quantum logic, more precisely mathematics based on Takeuti´s quantum set theory. In references [2, 6, 8, 11, 13, 17] it was shown that the propositional systems of quantum local field theories [consisting of an infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space] are the characteristic structures of this mathematics (let we call it „quantum mathematics").

2.2.2 Then in the framework of this "quantum mathematics" one should construct [of course, apart from the special case when H is two dimensional, it is yet an open problem waiting for a solution (c.f. sections 3 and 4)] the set up of a computer in parallel to the von Neumann´s set up of the computer.

2.2.3 In quantum set theory the truth values of the sentences are evaluated by the quantum logic. Then in this approach the task of the computer should be to evaluate, to compute the truth values of the statements. In this way the task of the computing should be the formulation of statements (programing) and the evaluation of them by the computer (running the program). The "quantum processor" should be built on the quantum logic (of the basic Hilbert space[2]). So in this von Neumann´s meaning the computer based on the two valued logic (simply named it classical computer) mechanizes the mathematics based on the two valued logic, while the computer based on the quantum logic (let we call it here "quantum computer"[3]) should (could?) mechanize the mathematics based on the quantum logic, the "quantum mathematics".

2.2.4 Since in the "quantum mathematics" the real numbers defined by Dedekind's cuts are self-adjoint operators of the basic Hilbert space H [1], thus the "quantum real numbers" are self-adjoint operators and the algebra of them is the algebra of these operators. The binary numbers are replaced by the "quantum binary numbers", namely in symbols $(0, 1) \rightarrow (0, e(x), 1)$ [$e^2(x) = e(x)$, the orthogonal projector of the closed linear subspace x of H, i.e. x is an element of L]. Therefore in symbols: the machine-made code of a classical program has the form of (1, 0, 0, 1, 1, ….) then the machine-made code of a "quantum program" should (could?) have the form of (e(x), 1, 0, e(y), e(z), …, 0, ...).

2.2.5 One can find the illustration of the geometrical structure of the system´s local state space both in references [2, p. 198] and [6, p. 1059] and in Figure 1 above. Thus one might think of this structure as a „non commutative" Hilbert bundle. Then we arrive at the main result of this discussion:

The local states of the system [*consisting of an infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space*] are sections of the bundle. The time evolution of these local states is governed, instead of the global/total Hamiltonian, by the **local Hamiltonian**

2 Again, in this approach the basic Hilbert space is not restricted to a two dimensional state space. Naturally, in the special case when H is two dimensional, the quantum processor is also built on the quantum logic of the basic Hilbert space H.

3 We use the name quantum computer though, as we see, it is more general in principle than the quantum computer based on quantum bits.

of the system according to the eq. (5.8) in ref. [2] or eq. (30) in ref. [13]. This geometrical structure and time evaluation equations imply that:

*Proposition 2.3: D*ifferent alternatives [*for the individual members of the infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space*] given by an initial value of the evolution equation described by a section of the „non commutative" Hilbert bundle can be computed in *parallel*.

3. Thecnical details: local scalar fields of Bose-Einstein statistics

Let we consider the illustrative case of N real classically relativistic scalar fields of Lagrangian density (2.1). This system consists of an infinite collection of identical classical anharmonic oscillators of N degrees of freedom connected in space. Then the corresponding quantum field theory (QFT) should consist of an infinite collection of identical quantum anharmonic oscillators of N degrees of freedom connected in space.

Really, the alternative quantization method substitutes the individual members of the system by their quantum mechanical counterparts [2]. The *local state pace* H_A, as we saw in the foregoing section, is an A-valued Hilbert space (Hilbert A-module) of the form $L^2(\boldsymbol{R}^N)\otimes A$ [the tensor product of the complex separable Hilbert space $L^2(\boldsymbol{R}^N)$ and the C^*-algebra A of bounded operators of $L^2(\boldsymbol{R}^3)$]. In this approach the quantized system is described ***coherently*** because the algebra of bounded operators $B(H_A) = B(H)\otimes A$ of the local state space H_A is a ***factor*** [2, 8] (which means that we can not divide it into two factors).

Von Neumann's basic theorem of QM, namely that the canonical commutation relations (CCR's) have a unique solution up to unitary equivalence, has an extended form (see *Proposision 2.2)* in this framework: a B-irreducible set of unitary operators in the A-valued Hilbert space H_A satisfying the CCR's is uniquely determined up to A-unitary equivalence.

Again, as we noted above, this extension of von Neumann's theorem offers the possibility that one formulates QFT in terms of the A-valued Hilbert spaces in the same unique way, up to A-unitary equivalence as QM is formulated in terms of complex Hilbert spaces up to unitary equivalence [18].

The dynamics of the system is described by the unitary map

$$t \rightarrow exp\,(-iHt)$$

of H_A onto itself, where H is the **local Hamiltonian** of the system obtained by replacing the *Hamiltonian density* of the classical system with its operator counterpart one gets by the quantization algorithm.

$$H = H(\phi, \pi, \partial\phi) = 1/2\Sigma_{\alpha=1}^{N} [\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2] + V(\phi_1, \ldots, \phi_N),$$

where the fields ϕ_α and their canonical momentum densities π_α as operators in H_A satisfy the CCR's [2]. The classical equations of motion become well-defined operator equations in H_A and the local states (the ray's $\boldsymbol{\Phi}$ of H_A, i.e. for all $\Phi\in\boldsymbol{\Phi}$ we have $<\Phi|\Phi>_A = 1$, where $<|>_A$ denotes the A-valued inner product in H_A and 1 is the unity operator of A) are governed by the local Schrödinger equation [2]:

$$i\hbar\partial\Phi(t)/\partial t = 1/2\Sigma_{\alpha=1}^{N}[\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2]\Phi(t) + V(\phi_1, \ldots, \phi_N)\,\Phi(t),\ \Phi\in H_A \quad (3.1)$$

Then we arrived at the main result formulated in *Proposition 2.3.*

One can apply the **extension** of the perturbation theory of QM to solve this equation by using the interaction picture [2, 13]. The local Hamiltonian of the free fields is

$$H_0 = 1/2\Sigma_{\alpha=1}^{N}[\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2] = \Sigma_{\alpha=1}^{N}(N_\alpha + 1/2)p_0^\alpha,$$

where $p_0^\alpha = (\boldsymbol{p}^2 + m_\alpha^2)^{1/2}$, $\boldsymbol{p}^2 = (-i\partial)^2 = -\boldsymbol{\Delta}$, $\boldsymbol{\Delta}$ *is the Laplace operator, and* $N_\alpha = a_\alpha^+ a_\alpha$, a_α^+ is the creation while a_α is the annihilation operator in the local Fock space F_A of the free fields [2, 13].
p_0^α is the energy operator of a Klein-Gordon-like free particle of mass m_α, i.e. its Hamiltonian operator. In this framework the Haag-theorem does not block to solve the local Schrödinger equation for non-trivial interactions in the local Fock space F_A of the free fields [2, 13].
We apply this formalism to a system of Lagrangian density (2.1) considered localized in space to a cube with side-edges a. In that case the basic Hilbert space in the Takeuti's approach reduces to the Hilbert space $L^2([0,a]^3)$ of the square integrable functions over the domain of the cube.
We diagonalize p_0^α. It means the solutions of the eigenvalue equations

$$p_0^\alpha\phi_n = e^\alpha{}_n\phi_n, \quad \phi_n \in L^2([0,a]^3)$$

The wave functions of norm *1* have the form

$$\phi_{n1,n2,n3}(x,y,z) = (2/a)^3 1/2 \sin(n_1\pi/a)x \sin(n_2\pi/a)y \sin(n_3\pi/a)z \quad (3.2)$$

and

$$e^\alpha{}_{n1,n2,n3} = [\pi^2 a^{-2}(n_1^2 + n_2^2 + n_3^2) + m_\alpha^2]^{1/2},\ n_1, n_2, n_3 = 1, 2, 3, \ldots.$$

The quantized system localised it in a cube in space has a discrete energy spectra in the free field approximation.

3.1 Application in computing
The basic Hilbert space is $L^2([0,a]^3)$ spanned by the orthonormal functions of the relation (3.2) which set of functions constitutes a basis for this Hilbert space. L is the lattice of all closed linear subspaces of $L^2([0,a]^3)$ (the quantum logic of von Neumann). Then the totality of all L-valued functions provides the universe $V^{(L)}$ of Takeuti.
In the "quantum mathematics" based on $V^{(L)}$, the real numbers defined by Dedekind's cuts are self-adjoint operators of the Hilbert space $L^2([0,a]^3)$ as it was shown by Takeuti. Therefore the "quantum real numbers" are self-adjoint operators and the algebra of them is the algebra of these operators. The binary numbers are replaced by the "quantum binary numbers", in symbols $(0, 1) \rightarrow (0, p(X), 1)$ [$p^2(X) = p(X)$, the orthogonal projector of the closed linear subspace X of $L^2([0,a]^3)$, i.e. X is an element of L]. In this way we have in symbols:

> the machine-made code of a classical program has the form of (1, 0, 0, 1, 1, ….),
> then
> the machine-made code of a "quantum program" should have the form of ($p(X), 1, 0, p(Y), p(Z), \ldots, 0, \ldots$).

The unity operator *1* of $L^2([0,a]^3]$) belongs to the **true** logical value, the zero operator *0* belongs to the **false** logical value, while the projection operators $p(X), p(Y), p(Z), \ldots$ to the **true-false** values, e.g. $p(X)$ is **true** on the subspace X while it is false outside X (on the difference subspace $L\backslash X$). Clearly the number of the true-false values is **infinite.**
The local state space $H_A = L^2(R^N)\otimes A$ is isomorphic to the countably infinite direct sum $H_A = \Sigma_1^\infty \oplus A$ of the Hilbert A-module A. This means that we can represent H_A with infinite column vectors with operator entries from A. The local states are represented by the rays of norm *1* (the unity operator of A) in H_A. The expectation value of a local bounded observable F in the local state Φ in H_A is given by the formula

$$Exp\ F = <\Phi|F|\Phi>_A \in A$$

using the A-valued inner product of H_A.
The local Hamiltonian H is a real number valued function in $V^{(L)}$:

$$H = \Sigma_{[n]}\, E_n\, P(\phi_n)$$

where $P(\phi_n)$ is the orthogonal projector of the one dimensional subspace of $L^2([0,a]^3)$ spanned by the ray belonging to the eigenstate ϕ_n, while E_n is also a hermitian element of A from the spectrum of the local Hamiltonian (which is a hermitian operator in the A-valued Hilbert space H_A). Therefore:

$$E_n = \Sigma_{[m]}\, e_m P(\phi_m)$$

where the ordinary non-negative real number e_m is from the spectrum of E_n (which of course may has not only discrete but continuous spectrum, too).
Thus the local Hamiltonian of the quantized system is a hermitian valued function in $V^{(L)}$ with values of form

$$H(\Phi) = Exp\ H = \Sigma_{[n]}\, e_n P(\phi_n) \in A$$

Then one can express the expectation value of the local Hamiltonian H as a linear combination of binary number valued functions in $V^{(L)}$ having the form

$$b = \Sigma_{[n]}\, b(n) P(\phi_n),\ b(n) = 0 \text{ or } 1$$

where $P(\phi_n)$ is the orthogonal projector in $L^2([0,a]^3)$ belonging to the eigenstate ϕ_n. The set of these binary numbers is a subset of the set of all binary numbers in $V^{(L)}$.
Thus one can evaluate, in finite linear combinations, the evolution of the quantized system, localised in a cube, in the local Fock space F_A of the free fields by applying the eigenstates of the energy operators of the Klein-Gordon-like particles of mass m_α, and thus in a finite steps of recursions. Therefore one can approach (or at least estimate) the real numbers in the universe $V^{(L)}$ by linear combinations of "quantum binary numbers" in this Takeuti's universe.

4. Thecnical details: local spinor fields of Fermi statistics

Now we present the specific case of the fermions of spin ½ by using the Dirac equation and the standard notations (see ref. [19]). First let us see the mathematical formulation of the alternative quantization for fermions. This means that we study the theoretical method of solving the specific field theoretical example, in the framework of the alternative quantization [2], for Lagrangian density of the Dirac equation [19]

$$L(t,\boldsymbol{x}) = \{i/2[\psi^+\gamma^0\gamma^\mu\partial_\mu\psi - (\partial_\mu\psi^+)\gamma^0\gamma^\mu\psi] - m\psi^+\gamma^0\psi\}(t,\boldsymbol{x}),\ (t,\boldsymbol{x}) \in \boldsymbol{M^4}, \quad (4.1)$$

where γ denotes the Dirac matrixes [19] and $\boldsymbol{M^4}$ is the Minkowski space. With a simple calculation one can check that the classical equation of motion $\partial L/\partial\psi - \partial_\mu(\partial L/\partial\partial_\mu\psi) = 0$ gives really the Dirac equation:

$$(i\gamma^\mu\partial_\mu + m)\psi^* = 0 \qquad (4.2)$$

The *local state pace* H_A is an A-valued Hilbert space (Hilbert A-module) of the form $[\sum_1^4 \oplus L^2{}_i(\boldsymbol{R})]\otimes A$ [the tensor product of the complex separable Hilbert space $H = \sum_1^4 \oplus L^2{}_i(\boldsymbol{R})$ and the C^*-algebra A of bounded operators in $L^2(\boldsymbol{R}^3)$]. As we noted in the forgoing section, in this approach the quantized system is described ***coherently*** because the algebra of bounded operators $B(H_A) = B(H)\otimes A$ of the local state space H_A is a ***factor*** [2, 8].

The spectral decompositions of the Dirac spinor operators are as follow:

$$\psi = \int_{[p\sigma]} \{(2\varepsilon)^{-1/2}[a_{p\sigma}u_{p\sigma}exp(-ipx) + b_{p\sigma}^{+}u_{-p-\sigma}exp(ipx)\}\, dE_{p\sigma}, \qquad (4.3a)$$

$$\psi^*\gamma^0 = \int_{[p\sigma]} \{(2\varepsilon)^{-1/2}[a_{p\sigma}^{+}u^*_{p\sigma}\gamma^0 exp(-ipx) + b_{p\sigma}u^*_{-p-\sigma}\gamma^0 exp(ipx)\}\, dE_{p\sigma}, \qquad (4.3b)$$

where $\varepsilon = (\boldsymbol{p}^2 + m^2)^{1/2} = p_0$ and $E_{p\sigma}$ denotes a partition of the unity opertor of A. In accordance with the rule of the Fermi statistics, the creation and annihilation operators, $a_{p\sigma}^{+}$, $b_{p\sigma}^{+}$ and $a_{p\sigma}$, $b_{p\sigma}$, respectively, satisfy the anti-commutation relations

$$\{a_{p\sigma}, a_{p\sigma}^{+}\}_+ = 1, \qquad \{b_{p\sigma}, b_{p\sigma}^{+}\}_+ = 1 \qquad (4.4)$$

These operators are anti-commute in all other combinations.

The dynamics of the system is described by the unitary map

$$t \rightarrow exp\,(-iHt)$$

of H_A onto itself, where H is the **local Hamiltonian** of the system obtained by replacing the *Hamiltonian density* of the classical system with its operator counterpart one gets by the quantization algorithm. The Hamiltonian density of the classical system is

$$H = H(\psi, \pi, \partial\psi) = \pi\partial_0\psi - L \qquad (4.5)$$

From (4.1) one gets that $\pi = \partial L/\partial\partial_0\psi = (i/2)\psi^*$ and with a straightforward algebra using the Dirac equation (4.2), too, we obtain for the Hamiltonian density (4.5) that

$$H = \psi^*\boldsymbol{\alpha}(i\partial\psi) + [\beta m(i-1)/2]\psi^*\psi, \qquad (4.6)$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$ and $\beta = \gamma^0$. Then by using the relation $u^*_{\pm p\sigma}\gamma^0 u_{\pm p\sigma} = 2\varepsilon = 2(\boldsymbol{p}^2 + m^2)^{1/2} = 2p_0$ and the rules (4.4) we get from (4.6) for the spectral decomposition of the local Hamiltonian

$$H = \int_{[p\sigma]}(a_{p\sigma}^{+}a_{p\sigma} + b_{p\sigma}^{+}b_{p\sigma} - 1)p_0\, dE_{p\sigma} =$$

$$= \int_{[p\sigma}(N_{a,p\sigma} + N_{b,p\sigma} - 1)(\boldsymbol{p}^2 + m^2)^{1/2}\, dE_{p\sigma}. \qquad (4.7)$$

The classical equation of motion becomes well-defined operator equation in the local state space H_A and the local states (the rays $\boldsymbol{\Phi}$ of H_A, i.e. for all $\Phi \in \boldsymbol{\Phi}$ we have $\langle\Phi|\Phi\rangle_A = 1$, where $\langle|\rangle_A$ denotes the A-valued inner product in H_A and 1 is the unity operator of A) are governed by the local Schrödinger equation [2, 6] :

$$i\partial\Phi(t)/\partial t = [\int_{[p\sigma}(N_{a,p\sigma} + N_{b,p\sigma} - 1)p_0\, dE_{p\sigma}]\Phi(t), \quad \Phi \in H_A \qquad (4.8)$$

The energy component p_0 as an operator takes the form $p_0 = (\mathbf{p}^2 + m^2)^{1/2}$, $\mathbf{p}^2 = (-i\partial)^2 = -\Delta$, Δ is the Laplace operator, and the number operators $N_{a,p\sigma}$ and $N_{b,p\sigma}$ belong to the fermion of mass m and its anti-particle, respectively, in the local Fock space F_A [2]. We note that in this framework the Haag-theorem does not block to solve the local Schrödinger equation (4.8) for non-trivial interactions in the local Fock space F_A of the free fields [2].

4.1 Application in computing

If we apply the formalism of the foregoing section to a system of Lagrangian (4.1) localized in space to a box of cuboid form with side-edges a, b and c then the basic Hilbert space in the Takeuti's approach reduces to the Hilbert space $L^2([0,a],[0,b],[0,c])$ of the square integrable functions over the domain of the cuboid form. The local Hamiltonian operator of the free field has a diagonal form like equation (4.7) in the corresponding local state space $H_A = F_A$ where of course A is the C^*-algebra of operators in the Hilbert-space $L^2([0,a],[0,b],[0,c])$. It means that its eigenvalues are hermitian operators in $L^2([0,a],[0,b],[0,c])$. For example, in the lowest energy local state, in the local vacuum state $\boldsymbol{\Phi}_0$ when the local number operators equal to zero, the hermitian eigenvalue operator of the local free field Hamiltonian operator is

$$-p_0 \qquad (4.9)$$

It means that in the local vacuum state of the fermion this non vanishing term decreases the energy of the vacuum while we saw in the forgoing section that in the case of the scalar fields the lowest energy local state increases the energy of the local vacuum.

It is clear that the diagonalization of p_0 for this system under consideration is a special case of the one discussed in section 3. Therefore we can write from section 3 for the eigenstates of p_0 that

$$\phi_{n1,n2,n3}(x,y,z) = (8/abc)^{1/2} \sin(n_1\pi/a)x \sin(n_2\pi/b)y \sin(n_3\pi/c)z \qquad (4.10)$$

and for the eigenvalues that

$$e_{n1,n2,n3} = [\pi^2 (n_1^2/a^2 + n_2^2/b^2 + n_3^2/c^2) + m^2]^{1/2}, \quad n_1, n_2, n_3 = 1, 2, 3, \ldots\ldots \qquad (4.11)$$

So again, the physical system described by the Lagrangian density (4.1) and localised it in a cuboid form in space, after quantization, has a discrete energy spectra in the first approximation (in the free fields approximation). We know such a phenomenon from condensed matter physics. Namely the collective behaviour of the atoms in the quantum liquid He^3 at low temperature, below 3 K, can be described by the quasi-particle approach as it was showed by L. D. Landau and He^3 satisfies Fermi statistics, i.e. it behaves like fermions.

As to the application in computing one can simply repeat what we wrote in the section 3 above. Thus we can again conclude that one can evaluate, *in finite linear combinations*, the evolution of the quantized system of Lagrangian (4.1), localised in a cuboid form, in the local Fock space F_A of the free field of the fermion by applying the eigenstates (4.10) of the energy operator of the fermion of mass m, and thus in a finite steps of recursions. Therefore one can approach (or at least estimate) the real numbers in the universe $V^{(L)}$ of Takeuti by linear combinations of "quantum binary numbers" in this Takeuti's universe. Then we can conclude that a physical system having eigenstates of form (4.10) can help us to solve the the system's evolution equation of form (4.8) by exciting it and measuring its eigenstates and the corresponding eigenvalues while inserting the results in the appropriate mathematical relations.

Therefore the cuboid form of a “well and appropriately tuned up” rigid or condensed body, of our examples in this section, may be an essential part of the physical implementation of the processor for a “quantum computer” of this type.
It is an important observation that the initial values of the evolution equations (3.1) and (4.8) are generated by the lattice of the projectors of the basic Hilbert space $L^2([0,a],[0,b],[0,c])$ which is an infinite set. This fact has a serious consequence in cyber security and cryptography based on the quantum computers discussed in this paper.

As a closing note of this section we remember again that, as it is well known, L. D. Landau described the quantum liquids He^3 and He^4 at low temperature (below 3 K) by applying the quasi particle approach. He called the quasi particles (elementary excitations) as „rotons”. Thus these physical materials and rotons may be the candidates for building the processors of “quantum computers” of this type.

5. Conclusions

5.1) In this rewiev paper we presented the foundations and resesults of parallel computing in terms of Gaisi Takeuris’s quantum set theory [1]. We saw that this approach is based on 1) the extension of the von Neumann’s uniqness theorem for the solutions of the canonical commutations relations in quntum mechanics of finitely dergrees of freedom to quantum local field theories of infinitely many degrees of freedom [2]; 2) the result of the mathematician Gaisi Takeuti about quantum set theory in extension theory; and the results of the physicist Lev Davidovics Landau about the behavior of the quantum liquids He^3 and He^4 resulting in a more general approach than the one base on the notion of the quantum bit.
5.2) One should construct (of course, apart from the special case when H is two dimensional) the set up of this type of a computer in parallel to the von Neumann´s set up of the binary computer. Therefore as a corollary this approach could and should offer a computing machinery exceeding the capacity of the computers we are using in this decades.
5.3) A physical system having eigenstates of form in the relation (3.2) or in (4.10) can help us to solve the system’s evolution equation by exciting it and measuring its eigenstates and the corresponding eigenvalues while inserting the results in the appropriate mathematical relations. Therefore the cube of a “well and appropriately tuned up” rigid or condensed body may be an essential part of the physical implementation of the processor for a “quantum computer” of this type.
5.4) L. D. Landau described the quantum liquids He^3 and He^4 at low temperature (below 3K) by applying the quasi particle approach outlined in sections 3 an 4. He called the quasi particles (elementary excitations) as “rotons”. Thus these physical materials and rotons may be the candidates for building the processor of **a “quantum computer” based on quantum set theory offering a more general framework than the one based on the notion of the quantum bit.**
5.5) The sets of inicial values of the local Schrödinger equations (3.1) and (4.8) are **infinite** sets in the corresponding local state spaces H_A’s. *This fact should play a decesive role in ciber security and criptography.*

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